Accommodating the source (and receiver) array in free-surface multiple elimination algorithm: impact on interfering or proximal primaries and multiples

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SUMMARY

Free-surface multiple elimination (FSME) algorithm (Carvalho, 1992; Weglein et al., 1997) is modified and extended to accommodate a source (and receiver) array with a radiation pattern. That accommodation can provide added value compared to previous methods that assumed a single point source (air-gun) for the fidelity of amplitude and phase prediction of free surface multiples at all offsets. For the source-array data, if all prerequisites are provided, the new algorithm has the theoretical capability of predicting the exact phase and amplitude of multiples, and in principle removing them through a simple subtraction. Green's theorem method can provide all its data requirements: (1) removing the reference wavefield, (2) estimation of source wavelet and radiation pattern, and (3) source and receiver deghosting. Green's theorem method is consistent with the new FSME algorithm. They are both multidimensional and do not require any subsurface information. The new FSME algorithm is tested on a 1D acoustic model, and the results indicate that the new algorithm enhances the multiple prediction when the data and experiment are caused by an array rather than a single air-gun.

INTRODUCTION

In marine seismic exploration, multiple removal is a classic long-standing problem. Various methods (e.g., Carvalho, 1992; Verschuur et al., 1992; Weglein et al., 1997, 2003; Berkhout and Verschuur, 1999; Dragoset et al., 2008) have been developed to either attenuate or eliminate free-surface multiples, and each method has different assumptions, advantages, and limitations. Among these methods, the inverse scattering series (ISS) FSME method (Carvalho, 1992; Weglein et al., 1997) does not need any subsurface information, which is a big advantage, especially under conditions of complex geology. The ISS method predicts the free-surface multiples accurately, while the feedback-loop method (Verschuur et al., 1992) only provides approximate predictions because it ignores the obliquity factor and retains the source-side ghost. Therefore, the ISS method can remove the free-surface multiples through a simple subtraction, and most importantly it preserves primary energy (e.g., Carvalho, 1992; Araújo, 1994; Weglein et al., 1997), while the feedback-loop method has to remove the multiples adaptively using certain criteria (energy minimization, for example). The energy minimization criterion works well when there are no overlapping or proximal primaries and multiples in the input data. If primaries and multiples are overlapping and destructively interfering, the energy minimization criterion can be invalid or fail and the adaptive subtraction will not work very well.

To predict free-surface multiples precisely, the ISS method has certain data requirements: (1) removal of the reference wavefield, (2) an estimation of the source wavelet and radiation pattern, and (3) source and receiver deghosting. Green's theorem wave separation methods that are consistent with the ISS method have been applied to provide these three criteria, since they are both multidimensional wave theoretic preprocessing methods and do not need any subsurface information. Green's theorem methods offer a flexible framework for deriving a number of useful algorithms due to the freedom of choosing a reference medium. When choosing air-water as the reference medium, the reference wavefield and the scattered wavefield can be seperated, and the source wavelet and radiation pattern can be estimated (Weglein and Secrest, 1990; Weglein et al., 2002). When choosing the whole space of water as the reference medium, the ghosts can be removed. Green's theorem methods have been pioneered by J. Zhang (Weglein et al., 2002; Zhang and Weglein, 2005, 2006; Zhang, 2007) and developed by J. Mayhan (Mayhan et al., 2011, 2012; Mayhan and Weglein, 2013). If all the prerequisites are provided, Zhang (2007) has shown that the ISS FSME algorithm can predict free-surface multiples accurately for a point-source data and remove them from the data without the need of adaptive subtraction.

However, for source-array data, the ISS FSME algorithm is not sufficient because this method assumes a single point source. In other words, the source has no variation of amplitude or phase with take-off angle. Nevertheless, in towed marine acquisition, a source array is commonly used to increase the power of the source, broaden the bandwidth, and cancel the random noise. The source array exhibits directivity in take-off angle (Loveridge et al., 1984). That directivity is an issue for AVO analysis and removing or attenuating multiples. In seismic processing, it is essential that we characterize the source (and receiver) array's effect on any seismic processing methods. Therefore, to improve the accuracy of the predicted multiples, the ISS FSME algorithm is extended by accommodating a source array. That accommodation can enhance the fidelity of amplitude and phase prediction of free surface multiples at all offsets.

THEORY

The ISS FSME algorithm is a fully data-driven algorithm and does not require any subsurface information. It has the ability to accurately predict the free-surface multiples order-by-order and then remove them through a simple subtraction. The ISS FSME algorithm for an isotropic point source in a 2D case is given by (Carvalho, 1992; Weglein et al., 1997, 2003):

$$D'_n(k_g,k_s,\omega) = \frac{1}{i\pi A(\omega)}\int dk D'_1(k_g,k,\omega) q e^{iq(\varepsilon_g+\varepsilon_s)} D'_{n-1}(k,k_s,\omega), \quad (1)$$

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where k_g, k_s and ω represent the Fourier conjugates of receiver, source, and time, respectively. ε_g and ε_s are the receivers' and sources' depth below the free surface, respectively. The obliquity factor q is given by $q = sgn(\omega)\sqrt{\omega^2/c_0^2 - k^2}$, and c_0 is the reference velocity. The FSME algorithm only requires the source signature $A(\omega)$ and source and receiver side deghosted data $D'_1(k_g, k, \omega)$ as its input. The free-surface multiples are predicted order-by-order and then added together give the deghosted and free-surface demultipled data $D'(k_g, k_s, \omega) = \sum_{n=1}^{\infty} D'_n(k_g, k_s, \omega)$.

For source-array data, the ISS FSME algorithm can only predict multiples approximately. To incorporate the source array, the FSME algorithm is extended from a single point source to a source array with a radiation pattern, as follows:

$$\begin{aligned} & D'_n(k_g, k_s, \omega) \\ & \quad = \frac{1}{i\pi}\int \frac{dk}{\rho(k,q,\omega)} D'_1(k_g, k, \omega) q e^{iq(\varepsilon_g+\varepsilon_s)} D'_{n-1}(k, k_s, \omega), \quad (2) \end{aligned}$$

where $\rho(k,q,\omega)$ is the projection of source signature in the f-k domain and $k^2 + q^2 = \omega^2/c_0^2$. The projection of source signature $\rho(k,q,\omega)$ can be directly achieved from the reference wavefield that is separated from the measured data by using Green's theorem method (Weglein and Secrest, 1990) by choosing air-water as its reference medium.

The key point is to obtain the projection of source signature $\rho(k,q,\omega)$ from the reference wavefield. We assume that the source array is invariant from one shot to the next. In other words, the geometry or the distribution of the source array remains for each shot. The direct reference wavefield P_0^d for a 2D case can be expressed as an integral of the direct reference Green's function G_0^d over all air-guns in an array,

$$\begin{aligned} & P_0^d(x, z, x_s, z_s, \omega) \\ & \quad = \int dx' dz' \rho(x', z', \omega) G_0^d(x, z, x' + x_s, z' + z_s, \omega), \quad (3) \end{aligned}$$

where (x,z) and (x_s, z_s) are the prediction point and source point, respectively. (x', z') is the distribution of the source with respect to the source locator (x_s, z_s). Using the bilinear form of Green's function and Fourier transforming over x, we obtain the relationship between ρ and P_0^d as

$$P_0^d(k, z, x_s, z_s, \omega) = \rho(k, q, \omega) \frac{e^{iq|z-z_s|}}{2iq} e^{ikx}. \quad (4)$$

Since $k^2 + q^2 = \omega^2/c_0^2$, q is not a free variable, hence, we can not obtain $\rho(x,z,\omega)$ in space-frequency domain by taking an inverse Fourier transform on $\rho(k,q,\omega)$. However, the projection of the source signature $\rho(k,q,\omega)$ can always be achieved directly from the direct reference wavefield P_0^d in the f-k domain, where the variable k or q represent the amplitude variations of the source signature with angles. Ikelle et al. (1997) also proposed a similar quantity $A(k,\omega)$, the inverse source wavelet, and solved it indirectly using the energy minimization criterion.

Substituting the projection of the source signature $\rho(k,q,\omega)$ into the inverse scattering free-surface removal subseries, the new FSME algorithm (equation 2) can be derived (Yang and Weglein, 2012). The new algorithm accommodates a source (and receiver) array and can provide added value for the fidelity of amplitude and phase prediction of free surface multiples at all offsets. The new FSME algorithm is fully multidimensional and does not require any subsurface information. Therefore, it is consistent with Green's theorem methods that provide all the data requirements. The new FSME algorithm (equation 2) is also consistent with the previous FSME algorithm (equation 1) when the source array reduces to a point source.

NUMERICAL TESTS

In this section, we will show numerical tests of the free-surface multiple removal for the source-array data with overlapping or interfering primaries and multiples. The numerical tests are based on a 1D acoustic model with varying velocity and constant density, as shown in Figure 1. The model has one shallow reflector at 90m, hence, the primary is interfering and overlapping with the free-surface multiples. The depths of the source and receiver are 13m and 18m, respectively. Using the Cagniard-de Hoop method, the synthetic data are generated by a source array (Figure 2) that contains nine air-guns in one line with 24m range. The advantage of the Cagniard-de Hoop method is that we can accurately calculate any specific event we are interested in, so that we can compare it with the results predicted by the FSME algorithm. Here, we assume that the source array only varies laterally with identical source signatures, but the assumption is not necessary in the ISS FSME theory.

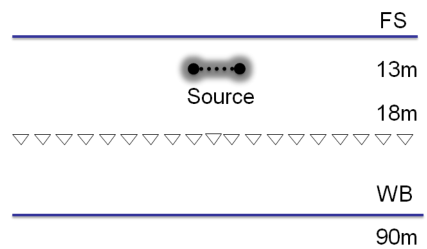

Figure 1: One-dimensional acoustic constant-density medium.

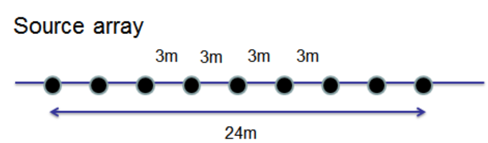

Figure 2: Source array with nine air-guns.

The tests are organized as follows: We first preprocess the generated source-array data using Green's theorem methods. After data preprocessing, we input the data into the previous FSME (equation 1) and the new FSME (equation 2) algorithms to predict and remove free-surface multiples and compare their results.

Data preprocessing by using Green's theorem methods

Figure 3(a) illustrates the data set generated by a source array with nine air-guns using the Cagniard-de Hoop method.

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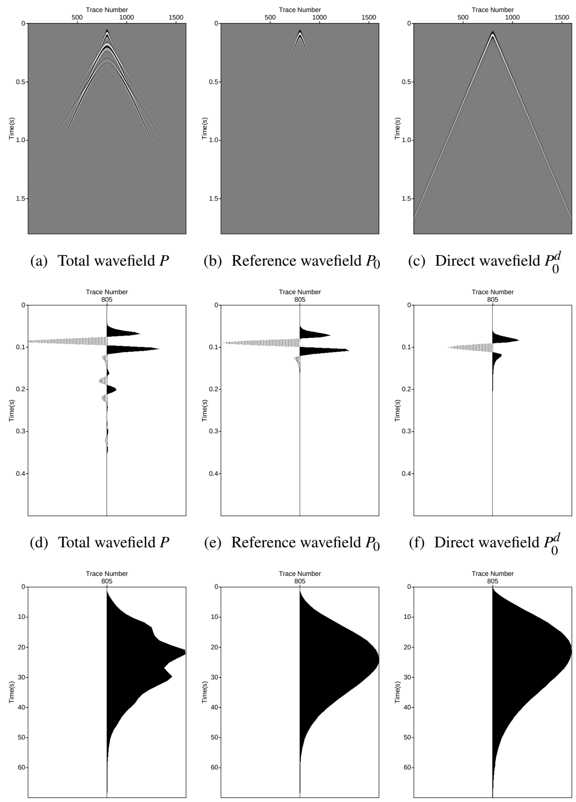
For simplicity, only the primary and the first-order free-surface multiple and their corresponding ghosts are generated. As we discussed above, Green's theorem methods are consistent with the new FSME method, because they are multidimensional and do not require any subsurface information. Furthermore, Green's theorem methods do not care about the source distribution, hence, the source-array data can be preprocessed by Green's theorem methods to satisfy the data requirement of the FSME algorithm. When choosing the air-water as the reference medium, Green's theorem wave separation method separates the total wavefield P (Figure 3(a)) into two parts: the reference wavefield P_0 (Figure 3(b)) and the scattered wavefield P_s (Figure 4(a)). After wave separation, Green's theorem deghosting method is needed to deghost the reference wavefield and the scattered wavefield by choosing the whole space of water as its reference medium. Figure 3(c) shows the direct reference wavefield P_0^d by deghosting the reference wavefield P_0. It can be seen that most of far offset energies are recovered. Figures 3(d), 3(e), and 3(f) represent the wiggle plots of the zero-offset traces. We can see that the reference wavefield is separated and its ghost is removed very well. From the spectra plots, we can see that the low frequency information is boosted, as shown in Figures 3(g), 3(h), and 3(i). Figures 4(b) and 4(c) illustrate the scattered wavefield P_s after removing the receiver-side ghosts and source & receiver ghosts, respectively. Figures 4(d), 4(e), and 4(f) are the wiggle plots of the zero-offset traces and Figures 4(g), 4(h), and 4(i) are their corresponding spectra plots. The notch at $c_0/2d = 1500/(2*18) \approx 42Hz$ is removed after receiver side deghosting. Both receiver side deghosting and source side deghosting recover more low frequency information and do not touch the primary.

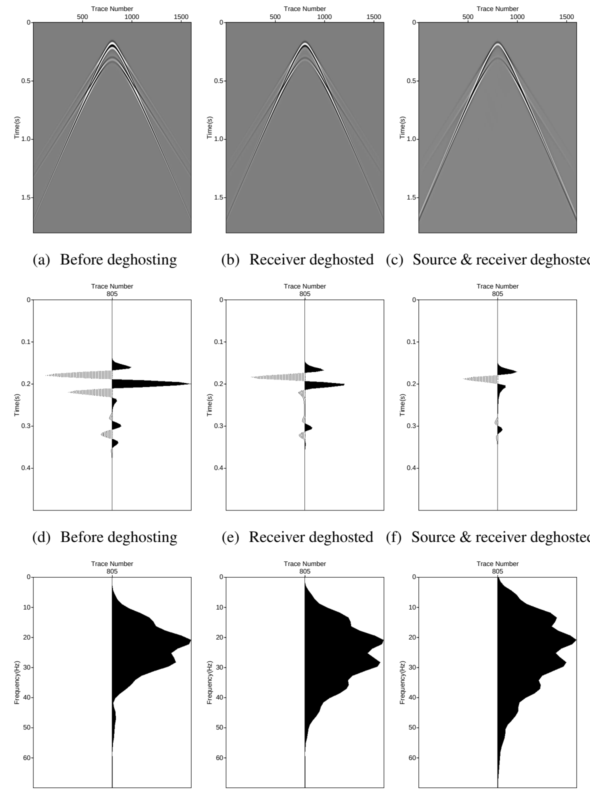


(a) Total wavefield P (b) Reference wavefield P_0 (c) Direct wavefield P_0^d

(d) Total wavefield P (e) Reference wavefield P_0 (f) Direct wavefield P_0^d

(g) Total wavefield P (h) Reference wavefield P_0 (i) Direct wavefield P_0^d

Figure 3: Wave separation and deghosting



(a) Before deghosting (b) Receiver deghosted (c) Source & receiver deghosted

(d) Before deghosting (e) Receiver deghosted (f) Source & receiver deghosted

(g) Before deghosting (h) Receiver deghosted (i) Source & receiver deghosted

Figure 4: Deghosting the scattered wavefield

Free-surface multiple removal

With all data requirements satisfied, we input them into the previous FSME (equation 1) and the new FSME (equation 2) algorithms to predict and remove free-surface multiples and compare their results. The source and receiver side deghosted data (Figure 4(c)) are replotted in Figure 5(a) to show more details. Figure 5(b) is its corresponding wiggle plot for a small window (times from 1.0s to 1.4s and traces from 1330 to 1420); we can see that the primary and the first-order free-surface multiple are overlapping when the offset exceeds approximately 1000m. Furthermore, in Figure 5(b) it can be seen that they are destructively overlapping. Therefore, the adaptive subtraction method can be invalid or fail for this kind of situation, because the method is based on the energy minimization criterion, which assumes that the energy of the data will be minimized after the multiples are removed. However, in this case, the energy increases after removal of the multiples.

First, we apply the previous FSME algorithm (equation 1) to predict free-surface multiples. It predicts phase accurately but an approximate amplitude. After removing the free-surface multiple, Figure 5(c) shows that most multiples are removed,

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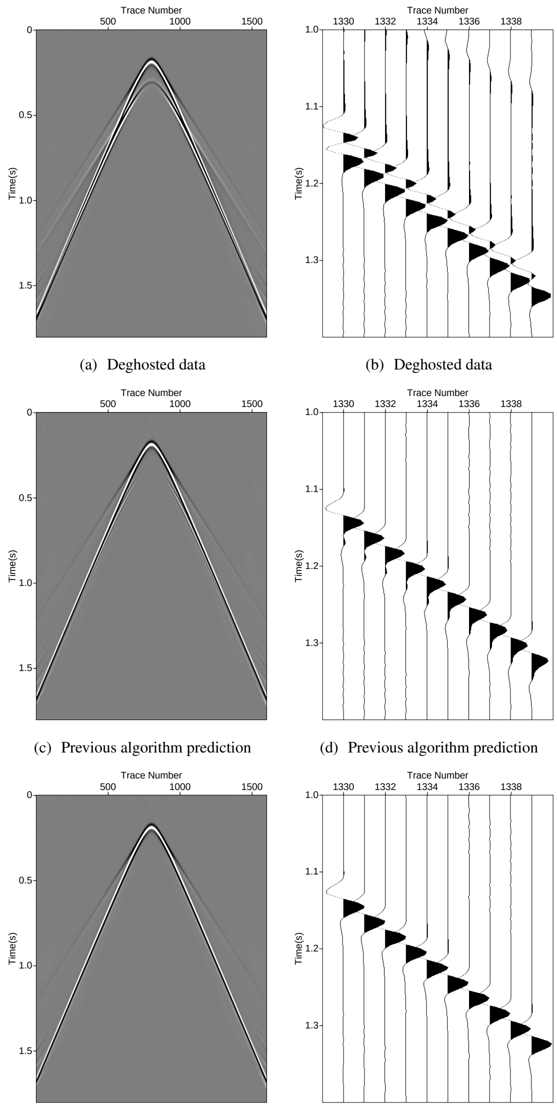
but there are still some residual multiples. Whether this result is valuable or not depends on the objective. If the amplitude is not critical, then this method is sufficient. For cases like AVO analysis and inversion, in which amplitude is important, such residual multiples could produce errors in the prediction.



(a) Deghosted data (b) Deghosted data

(c) Previous algorithm prediction (d) Previous algorithm prediction

(e) New algorithm prediction (f) New algorithm prediction

Figure 5: The left column are the input data and after free-surface multiple removal using the previous and new FSME algorithms. The right column are their corresponding wiggle plots for a small window (times from 1.0s to 1.4s and traces from 1330 to 1420).

Next, the new FSME algorithm (Equation 2) is used to predict free-surface multiples. It can predict both amplitude and phase accurately for the source-array data at all offsets. After a simple subtraction, all the multiples are eliminated completely, as shown in Figure 5(e). Therefore, the new FSME algorithm works very well for the source-array data that have interfering events. Comparing Figures 5(f) and 5(d), we can see that the primary is still affected by the residual multiple in Figure 5(d), while in Figure 5(f), the primary remains untouched as the original primary. Figure 6 illustrates the detail of comparison for one trace at offset $= 1800m$. After removing

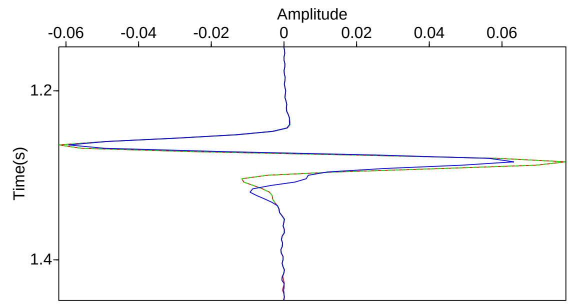


Figure 6: Red: the original primary in the input data; Blue: after multiple removal using the previous FSME algorithm; Green dash: after multiple removal using the new FSME algorithm.

free-surface multiple using the new algorithm, the primary is the same as the original one in the input data, while using the previous algorithm, the primary (Figure 5(d)) is a little weaker than the original primary, and this amplitude error will seriously affect AVO analysis.

CONCLUSIONS

A new FSME algorithm is proposed and tested on source-array data that have interfering primaries and multiples. The new FMSR algorithm accommodates a source (and receiver) array and can provide added value compared to previous methods for the fidelity of amplitude and phase prediction of free surface multiples at all offsets. If all prerequisites are provided, the new FSME algorithm, in principle, has the ability to predict free-surface multiples precisely, and removing them through a simple subtraction. All prerequisites can be achieved using Green's theorem methods by choosing different reference media. The new FSME algorithm is consistent with Green's theorem methods. They are both multidimensional and do not need any subsurface information. The numerical tests show that for source-array data, the previous isotropic source FSME algorithm can only predict phase accurately but amplitude approximately. This amplitude error can seriously affect the prediction results, such as AVO analysis and inversion, when a multiple intersects a primary. The new FSME algorithm could accommodate array data and eliminate free-surface multiples without damaging primaries.

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